A classification of syllogistics with simple positive terms[1]

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Abstract. *In this paper are given the account of a classification of some syllogistics with simple common terms (among them are appropriate fragments of logics of Aristoteles (C2), of Leibniz (ФС), of Lewis Carroll (КС), of Bolzano (БС), fragment of traditional logic (syllogistic of Łukasiewicz, C4) and some another theories). Results are presented in graph (its points corresponding to syllogistics and connectives between points corresponding to the relation of inclusion between the sets of the theorems of the syllogistics).*

В данной статье описывается построение классификации нескольких теорий чистой позитивной силлогистики (теорий C2, ФС, КС, БС, C4, C3 и C3.1). В основе классификации лежит задание на множестве всех теорий, сформулированных в одном языке на базе классической логики, отношения порядка. Результаты классификации представлены в виде направленного графа. Указан ряд синтаксических расширений и ослаблений классических силлогистических теорий.

1. Method of classification. For building the classification I modified a method, which had been used by A.S. Karpenko for classifying some propositional logics (in English see, e.g., [Karpenko 1991, 1992, 1993]). The method applied by me may be described in the following way.

1) The theories being classified are formulated in common language.
2) The sets of deductive postulates (rules and axioms) of the theories are formed in such a way, that all the theories look like conservative extensions of a minimal system, called *the basis (of the classification).*
3) On the set of the theories formulated in such a way we determine a relation of order.
4) We look for upper and lower bounds (preferably exact ones) of the set of the theories being classified and of its different subsets.
5) The analytical purpose is to reconstruct of the lattice of the theories (however, it was not my purpose for the present research).
6) Practical purpose of the work to construct a graph representing the created classification. The practical value of such graph is facilitate understanding of the correlations between analysed theories.

2. Initial definitions.

1) *A (formal) theory* is a set of formulae closed concerning rules of deduction.
2) We will refer to the set of the theories being classified as to the set *H*.
3) The language of the theories consists of:
   a) *logical constants*: $\rceil$, $\wedge$, $\vee$, $\supset$, $\equiv$;
   b) *syllogistic constants*: a, e, i, o (we consider them as two-valence predicates of the first order);
   c) infinite list of *syllogistic terms*: S, P, M, S_1; P_1, M_1, … (we consider them as *individual terms*);
   d) round brackets.

The formula:

a) an expression looks like $\alpha_1*\alpha_2$, where α_1 and α_2 are individual constants and * is a syllogistic constant;

b) an expression correctly formed from other expressions with assistance of logical constants.

4) As a basis of classification I will fix a formulation of the classical propositional logic adapted to the using language. All other systems will be constructed adding axioms to the basis.

5) The theory corresponding to the basis I will designate $C_\varnothing$ and its conservative inconsistent extension (the set of all formulae of the language) – $C_\perp$.

6) If T_1 and T_2 are formal theories in the language and $Cn(\Gamma)$ – deductive closure of set Γ, then

a) T_1 less, then $T_2 \Leftrightarrow T_1 \subseteq T_2$;

b) $\min(T_1,T_2)=T_1 \cap T_2$;

c) $\max(T_1,T_2)=Cn(T_1 \cup T_2)$.

If $T_1 \subseteq T_2$, then T_1 will designate as *sub-theory of* T_2.

If I add axioms A_1, A_2, … to the axiomatics of T, then I will write: $T+A_1+A_2+\ldots$ (or $T+\{A_1, A_2, \ldots\}$), and, if I remove the axioms, I will write: $T-A_1-A_2-\ldots$ (or $T-\{A_1, A_2, \ldots\}$).

3. The set H of the systems being classified. I considered eight syllogistics with common simple terms. These are appropriate fragments of logics of Aristoteles (C2), Leibniz (fundamental syllogistics, ФС), Bolzano (БС), Carroll (КС), traditional syllogistics (syllogistics of Łukasiewicz, C4), and systems C1, C3, C3.1. Formulations of the theories are taken from [Markin 1991] with some alterations. In table 1 it is pointed, which axioms we have to add to $C_\varnothing$ in order to construct one or another theory from H. The sign "+" in the table means that the formula of the line is an axiom of the system pointed in the column; "|–" – that the formula is not an axiom, but it is a theorem of the system; "–" – that the formula is not a theorem.

Table 1.

N	**Axioms**	**C1**	**C2**	**КС**	**БС**	**C3**	**C3.1**	**ФС**	**C4**
1	$(SaM \wedge MaP) \supset SaP$	+	+	+	+	+	+	+	+
2	$(SaM \wedge MeP) \supset SeP$	+	+	+	+	+	+	+	+
3	$SiP \supset PiS$	+	+	+	+	+	+	+	+
4	$SaP \supset SiP$	+	+	+	+	+	+	–	+
5	$SiP \supset SiS$	–	+	+	+	|–	|–	+	|–
6	$SiS \supset SaS$	–	+	+	+	–	–	+	+
7	$SaP \supset (SaS \wedge PaP)$	–	|–	|–	|–	–	+	|–	|–
8	$SoP \supset SiS$	–	–	|–	|–	+	|–	+	+
9	$SeP \supset SaS$	–	–	–	|–	–	+	+	|–
10	$SeP \equiv \rceil SiP$	+	+	+	–	+	+	+	+
11	$SeP \equiv \rceil SiP \wedge SiS$	–	–	–	+	|–	|–	–	|–
12	$SoP \equiv \rceil SaP$	+	+	–	–	+	+	+	+
13	$SoP \equiv \rceil SaP \wedge SiS$	–	–	+	+	|–	|–	|–	|–

4. Upper and lower bounds of H and of its subsets. It is clear from table 1, that C4 is supremum of H. I describe below some lower bounds of H and of some of its subsets (theories C, C_0, $C_{0.1}$, $C_{0.2}$, $C_{0.3}$). For the description of the new theories and the theories from H in an uniform way I had to change some of the axioms being used. In the new variants of formulation the formulae 10-13 from table 1 are substituted by deductively weaker ones. The results of the reformulation are produced in table 2.

Table 2.

N	**Axioms**	**C1**	**C2**	**КС**	**БС**	**C3**	**C3.1**	**ФС**	**C4**
1 – 7	formulae – as in the table 1.								
8	$SoP \supset SiS$	–	–	+	+	+	|–	+	+
9	$SeP \supset SaS$	–	–	–	+	–	+	+	|–
14	$\neg SiP \wedge \neg SiS \supset SeP$	+	+	+	–	+	+	+	+
15	$\neg SaP \wedge \neg SiS \supset SoP$	+	+	–	–	+	+	+	+
16	$SeP \supset \neg SiP$	+	+	+	+	+	+	+	+
17	$\neg SiP \wedge SiS \supset SeP$	+	+	+	+	+	+	+	+
18	$SoP \supset \neg SaP$	+	+	+	+	+	+	+	+
19	$\neg SaP \wedge SiS \supset SoP$	+	+	+	+	+	+	+	+

Formulae 1-3 and 16-19 determine the theory C which is a lower bound of H. Below are placed some subsets of H (to the left) and some of their lower bounds (to the right).

{C1, C2, C3, C3.1, КС, БС}	$C_0 = C+(SaP \supset SiP)$
{C2, КС, БС}	$C_{0.1} = C_0 + SiP\supset SiS + SiS\supset SaS$
{C2, КС}	$C_{0.2} = C_{0.1} + \neg SiP\wedge \neg SiS\supset SeP$
{КС, БС}	$C_{0.3} = C_{0.1} + SoP\supset SiS$

5. Graph of syllogistics. Using the data shown above, I made a graph (figure 1) showing correlations between the theories. Points of the graph correspond to syllogistic theories, and connections between them correspond to the given relation of order.

There are also some extensions of C4 represented in the graph:

1) $C_{=}$=C4+$SiP\supset SaP$;
2) C(2)=$C_{=}$+$SeM\wedge MeP\supset SaP$.

I will tell about them in next two paragraphs.

C4 is not a minimal system completed to $C_{=}$ by joining ($SiP\supset SaP$), since ($SiP\supset SaP$) already give $C_{=}$ with C3 and with C3.1.

There are some theories between C4 and $C_{=}$, e. g.: (C4 + $SaP\supset PaS$), (C4 + $MiP\wedge SiM\supset SiP$), (C4 + $SaP\supset PaS$ + $SiM\wedge MiP\supset SiP$). In the first of them predicate “a”, in the second – “i”, in the third – both of the predicates express relations of equivalence, in the last theory these relations are connected with the law of subordination ($SaP\supset SiP$).

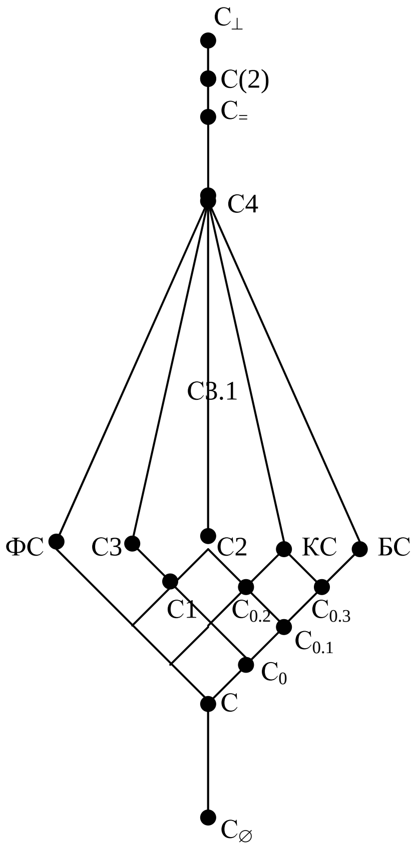

Figure 1.

6. Theory $C_=$ and syntactic consistence. In figure 1 the theory $C_=$ is placed above C4. This theory shows us two aspects of interrelations between the theories investigated. Firstly, $C_=$ points out correlation between syllogistic relations a, i, e, o (considered as relations of set theory) and usual extensional relations on sets. In $C_=$ predicates a and i are equivalent and represent some relations of equality of extents. Predicates e and o are negations of a and i. Secondly, it is easy to demonstrate syntactic consistence of the theories of H by referring to $C_=$. Let's consider the second aspect in more detail.

Lemma 1. $C4+SiP \supset SaP = C_\varnothing+\{SaS, SaP \supset PaS, SaM \wedge MaP \supset SaP, SiP \equiv SaP, SeP \equiv \rceil SiP, SoP \equiv \rceil SaP\}$.

MT.1. $C_=$ is syntactically consistent.

Sketch of proof. Theory $C_\varnothing+\{SaM \wedge MaP \supset SaP, SaS, SaP \supset PaS\}$ is syntactically consistent. Otherwise, every theory which has the first one as its subtheory would be syntactically inconsistent. Obviously, addition of axioms $SiP \equiv SaP$, $SeP \equiv \rceil SiP$ and $SoP \equiv \rceil SaP$ to this theory does not make the theory inconsistent, too.

Corollary from MT.1. Every subtheory of $C_=$ is syntactically consistent.

7. Syntactic incompleteness of theories of H. Theory C(2). Existence of $C_=$ reveals syntactic incompleteness of theories of H (by pointing to unprovable in C4 and its subtheories formulae, e.g., $SiP \supset SaP$, $SaP \supset PaS$, $SiM \wedge MiP \supset SiP$). $C_=$ is not syntactically complete, too: neither SeP, nor

SaP conclude from SeM∧MeP in $C_=$, and this shows ways for subsequent syntactical extending of syllogistics. In figure 1 it is showed between $C_=$ and $C_\perp$ the theory C(2) determined as ($C_=$ + SeM∧MeP⊃SaP). I think that C(2) is syntactically complete extension of $C_=$.

8. Some more problems at the conclusion. Set of all theories, determined in common language with classical basis, and operations min, max and specific complement form Brouwerian algebra ([Smirnov 1987, p. 32-34] with reference to [Tarski 1956]). The set of such finitely axiomatizable theories with the same operations form Boolean algebra. Together with given order these sets form Brouwerian and Boolean lattices. Null and unity elements of these structures are the theories $C_\varnothing$ and $C_\perp$.

In continuation of the theme I would like also to point out the following tasks.

1. Building of minimal lattice with unity in C4 and with all theories of H among its elements. In other words, we have to find such independent axiomatics of C4 that formulations of the rest theories from H could be received by removal of certain axioms from the formulation of C4.
2. Estimating quantity of theories being between two comparable theories from the graph (e. g., between C1 and C2, C2 and C4, C4 and $C_=$).

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